Multilevel Krylov Methods

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Abstract

Here we developed a new type of multilevel methods, called multilevel Krylov method (MK method) to solve linear systems of equations. The basic idea of this type of methods is to shift small eigenvalues that are responsible for slow convergence, to a priori fixed constant. The shifting of the eigenvalues is similar to projection type methods and uses the solution of subspace or coarse level systems. Numerical results show that these MK methods work very well for convection-diffusion equations and the Helmholtz equation. The convergence can be made almost independent of grid size h and also only mildly dependent of the wavenumber k for the Helmholtz equation. We also combine the multilevel Krylov idea with the algebraic way of choosing the coarse-grid system and the restriction and prolongation operators. The resulting method is called algebraic multilevel Krylov method or AMK method. We present different AMK methods that differ in the choice of the algebraic components. First, we use the classical Ruge and Stüben approach. Then, we present a agglomeration-based technique. Both methods are tested for various matrices arising from discretization of 2D diffusion and convection-diffusion equation as well as for several matrices taken from the matrix-market collections. The numerical results show that the AMK methods work as well as the geometric MK methods. For the convection-diffusion equations the AMK methods lead to better convergence rates than the original AMG method by Ruge and Stüben.