Preconditioning Stochastic Saddle-point Problems

List of authors:

Catherine E. Powell [1]

Stochastic Galerkin and stochastic collocation methods are becoming increasingly popular for solving (stochastic) PDEs with random data (coefficients, sources, boundary conditions, geometry). The former usually require the solution of a large, coupled linear system of equations while the latter require the solution of a large number of small systems which have the dimension of the corresponding deterministic problem. Although many studies on positive definite problems with random data can be found in the literature, there is relatively little work on stochastic saddle-point problems.

In this talk, we give an overview of the linear algebra issues involved in applying stochastic Galerkin and collocation schemes to saddle-point problems with random data. We focus on a mixed formulation of a second-order elliptic problem and investigate the efficiency of preconditioning strategies of Schur-complement and augmented type, for use with MINRES. Obtaining preconditioners that are robust with respect to the spatial discretisation parameters, the choice of discretisation on the underlying probability space, as well as the statistical parameters of the random inputs is a tall order.

Stochastic collocation methods, when combined with suitable mixed finite element methods in the physical domain, yield standard deterministic saddle point systems. These are trivial to solve when considered individually; the challenge lies in exploiting their similarities to recycle information and minimize the cost of solving the entire sequence.

Galerkin approximations, which couple mixed finite element discretizations in physical space with global polynomial approximation on a probability space, also give rise to linear systems with familiar saddle point structure. However, the matrix blocks are sums of Kronecker products of pairs of matrices associated with two distinct discretizations and the systems are large, reflecting the curse of dimensionality inherent in most stochastic approximation schemes. For stochastically nonlinear problems, this is compounded by the fact that the matrices are block-dense and the cost of a matrix vector product is non-trivial.

[1] University of Manchester