# A Preconditioning Strategy Applicable To A Family Of IDR($s$) Methods For Reduction Of Computational Cost

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We consider to solve a large non-singular linear system of equations,

$$Ax = b$$

where $A$ is a given nonsymmetric coefficient $(N \times N)$-matrix, and $x$, $b$ are a solution vector and right-hand side vector of order $N$, respectively. Krylov subspace of order $n$ is spaned by

$$K_n(A; r_0) := \mathrm{span} r_0, Ar_0, \cdots, A^{n-1}r_0,$$

where $r_0(= b - Ax_0)$ is an initial residual vector. As known well, Krylov subspace methods are effective iterative methods for solving large linear systems of equations. Recently, IDR(s) method based on IDR (Induced Dimension Reduction) Theorem[4] was proposed by P. Sonneveld and M.B. van Gijzen[2]. Moreover, generalization and extension of IDR Theorem was studied by G. Sleijpen[1] and M. Tanio[3].

In a family of IDR($s$) methods, computation of multiplication of transpose matrix $P^T$ and a vector is needed commonly at every iteration. Here $P$ is a dense matrix in $R^{N\times s}$[2]. Computational cost of multiplication of $P^T$ and a vector dominates in the algorithm of several IDR($s$) methods, and the more parameter $s$, the more computational cost. Therefore it is an issue how we construct a structure of dense matrix $P$.

In this article, as one of preconditioning techniques, we propose a structure of a dense matrix $P$ composed with divided $s$ column vectors of size $\lfloor N/s \rfloor \times 1$. Here $\lfloor\rfloor$ denotes the so-called Gauss symbol. We refer to this partially dense matrix as Slim Dense (abbreviated as **S_Dense**) matrix. As a result of adoptation of this preconditioning strategy, computational cost of a family of IDR($s$) methods can be reduced.

Table 1 shows the convergence of GIDR($s$, $L = 2$) method for dense and S_Dense matrices $P$. In this Table, "$s_{opt.}$" means optimum parameter $s$ in view of CPU time. "*ave. time*" means average CPU time per one iteration in milli-seconds. "ratio" means ratio of CPU time of GIDR($s$, $L = 2$) method using S_Dense matrix to that of GIDR($s$, $L = 2$) method using dense matrices $P$. The bold figure means the least CPU time of GIDR($s$, $L = 2$) method of dense and S_Dense matrices $P$ for each matrix. From Table 1, we can see that the proposed preconditioning strategy using S_Dense matrix $P$ is very effective for improvement of efficiency of GIDR($s$, $L$) method.

References

[1] Sleijpen, G.L.G., Sonneveld, P., van Gijzen, M.B. : Bi-CGSTAB as an Induced Dimension Reduction Method, Depart. of Applied Math. Anal., TR08-07(2008), Delft University of Technology.

Table 1: Convergence of GIDR($s$, $L = 2$) method of dense and S_Dense matrices $P$.

| matrix | *ave. nnz* | structure of $P$ | $s_{opt.}$ | itr. | time [sec.] | ratio | *ave. time* [msec.] |
|---|---|---|---|---|---|---|---|
| 2*language | 2* 3.047 | dense | 1 | 44 | 1.19 | 1.0 | 27.068 |
| | | S_Dense | 3 | 40 | **1.07** | 0.901 | 26.825 |
| 2*epb3 | 2* 5.479 | dense | 2 | 2748 | 11.25 | 1.0 | 4.097 |
| | | S_Dense | 5 | 2424 | **9.91** | 0.881 | 4.090 |
| 2*memplus | 2* 7.104 | dense | 4 | 230 | 0.26 | 1.0 | 1.130 |
| | | S_Dense | 7 | 192 | **0.22** | 0.842 | 1.141 |
| 2*add20 | 2* 7.231 | dense | 3 | 152 | 0.027 | 1.0 | 0.178 |
| | | S_Dense | 6 | 140 | **0.025** | 0.926 | 0.179 |
| 2*matrix_9 | 2*20.512 | dense | 6 | 1960 | 25.30 | 1.0 | 12.910 |
| | | S_Dense | 6 | 1932 | **22.36** | 0.884 | 11.576 |
| 2*xenon2 | 2*24.556 | dense | 4 | 1450 | 32.36 | 1.0 | 22.318 |
| | | S_Dense | 5 | 1356 | **28.95** | 0.895 | 21.350 |
| 2*poisson3Db | 2*27.737 | dense | 4 | 380 | 6.35 | 1.0 | 16.721 |
| | | S_Dense | 5 | 360 | **5.81** | 0.915 | 16.144 |
| 2*sme3Dc | 2*73.344 | dense | 7 | 3456 | 6.76 | 1.0 | 17.870 |
| | | S_Dense | 8 | 3366 | **58.32** | 0.944 | 17.327 |
| 2*raefsky2 | 2*90.550 | dense | 8 | 378 | 0.35 | 1.0 | 0.931 |
| | | S_Dense | 8 | 378 | **0.33** | 0.957 | 0.892 |

[2] Sonneveld, P., van Gijzen, M.B.: IDR(s): a family of simple and fast algorithms for solving large nonsymmetric linear systems, SIAM J. Sci. Comput., Vol. 31, No.2, 1035-1062, 2008.

[3] Tanio, M., Sugihara, M., GIDR(s, L): Generalized IDR(s), Proceeding of the 2008 annual meeting of the JSIAM, pp.411-412, 2008.

[4] Wesseling, P., Sonneveld, P.: Numerical Experiments with a Multiple Grid-and a Preconditioned Lanczos Type Methods, Lecture Notes in Math., Springer, No.771, pp.543-562, 1980.