Dynamic Sparsity Pattern Based Factored Approximate Inverse Preconditioners For Indefinite Matrices

Eun-Joo Lee
Department of Computer Science, East Stroudsburg University of Pennsylvania, 327 Science and Technology Center, East Stroudsburg, PA 18301-2999, USA
elee@po-box.esu.edu
Jun Zhang
Laboratory for Computational Medical Imaging & Data Analysis Laboratory for High Performance Scientific Computing & Computer Simulation, Department of Computer Science, University of Kentucky, 773 Anderson Hall, Lexington, KY 40506-0046, USA
jzhang@cs.uky.edu

We propose two sparsity pattern selection algorithms for factored approximate inverse preconditioners on solving general sparse matrices. The sparsity pattern is adaptively updated in the construction phase by using combined information of the inverse and original triangular factors of the original matrix. In order to determine the sparsity pattern, our first algorithm uses the norm of the inverse factors multiplied by the largest absolute value of the original factors, and the second employs the norm of the inverse factors divided by the norm of the original factors. Experimental results show that these algorithms improve the accuracy and robustness of the preconditioners on solving general sparse matrices.

1 Introduction

The computational efficiency and potential parallelism of sparse approximate inverse (SAI) leads us to choose SAI preconditioners as an alternative to the conventional ILU preconditioners. It has been well known that the performance of SAI preconditioners depends on their sparsity patterns [4, 5]. Thus, obtaining an optimal sparsity pattern in constructing an SAI preconditioner can be the most important part.

For general sparse matrices, the prescribed sparsity pattern is often inadequate for robustness of SAI [5]. On the contrary, a dynamic sparsity pattern strategy adjusts the pattern using some rules in the construction phase. Such a strategy usually computes more accurate and robust preconditioners than a static strategy [4]. Thus, a dynamic sparsity pattern strategy may be used to substitute for a static sparsity pattern strategy in solving difficult matrices.

In recent years, a few dynamic pattern strategies [2, 3] for SAI preconditioners have been developed. For example, Bollhöfer [1] showed that the norms of the inverse triangular factors have direct influence on the dropping strategy in computing a new ILU decomposition. Based on this insight, Bollhöfer [2] proposed an algorithm that manages the process of dropped entries with small absolute values by using the row norm of any row of the inverse factors, but the algorithm has a limitation on solving some ill-conditioned problems.

As a part of our continuous efforts in determining dynamic sparsity pattern, we introduce two enhanced algorithms, which extend the algorithm [2] mentioned above, using combined information of the norm of the inverse factors and either the largest absolute value of the original factors or the norm of the original factors. Here, a factored approximate inverse (FAPINV) [5], which is a sparse approximate inverse with a factored form, is utilized as an SAI.

2. Dynamic sparsity pattern based factored approximate inverse preconditioners

We now introduce two algorithms that determine dynamic sparsity patterns for FAPINV preconditioners on solving general sparse matrices. In determining the sparsity pattern, we exploit information of the inverse and original factors by the following two supporting reasons that (1) the entries of FAPINV are computed by the original and previously computed inverse triangular factors [5], and (2) the norm of the inverse factor is strongly related with the dropping tolerance of FAPINV [1]. From that point of view, (1) our first algorithm, Norm-Largest-Dynamic sparsity pattern in Algorithm, computes FAPINV with the dynamic sparsity pattern using the norm of the inverse factors multiplied by the largest absolute value of the original factors, and (2) the second, Norm-Norm-Dynamic sparsity pattern Algorithm, employs the norm of the inverse factors divided by the norm of the original factors.

ALGORITHM: FAPINV WITH NORM-LARGEST-DYNAMIC SPARSITY PATTERN

1. Find the largest absolute values, $Large_L$ and $Large_U$, of the lower and upper parts of A
2. Do $j = n, 1$ with step (-1)
3. $\zeta_U(j) = \zeta_L(j) = 0$
4.   Do $i = j+1, n$
5.   Compute $U_{j,i}$ by using FAPINV [5]
6.   If $|(U_{j,i}| > \zeta_U(j))$, then $\zeta_U(j) = |U_{j,i}|$
7.   End Do
8. $\eta_U = \zeta_U(j) * Large_U$
9.   If $(\eta_U > 1)$, then $\tau = \tau/\eta_U$
10.   Do $i = j+1, n$
11.   If $(|U_{j,i}| < \tau)$, then $U_{j,i} = 0$
12.   End Do
13. $D_{j,j} = 1/(a_{j,j} + \sum_{k=j+1}^{n} U_{j,k} * a_{k,j})$
14.   Do $i = j+1, n$
15.   Compute $L_{i,j}$ by using FAPINV [5]
16.   If $(|L_{i,j}| > \zeta_L(j))$, then $\zeta_L(j) = |L_{i,j}|$
17.   End Do
18. $\eta_L = \zeta_L(j) * Large_L$
19.   If $(\eta_L > 1)$, then $\tau = \tau/\eta_L$
20.   Do $i = j+1, n$

21.   If $(|L_{i,j}| < \tau)$, then $L_{i,j} = 0$
22.   End Do
23. End Do

Note that $Large_L$ and $Large_U$ refer to the largest absolute values of the lower and upper triangular factors of the original matrix, respectively. The $\zeta_U(j)$ and $\zeta_L(j)$ in lines 6 and 15 denote the largest absolute values of the columns U_j^T and L_j , respectively. The upper inverse factor U and the lower inverse factor L are computed in lines 4–12 and lines 14–22, respectively, and the diagonal inverse D is constructed in line 13. In lines 5 and 15, $U_{j,i}$ and $L_{i,j}$ can be obtained by using the FAPINV algorithm [5]. In lines 9 and 19, the dropping tolerance τ is determined by η_U and η_L, and the value of τ is updated for each j. Finally, in lines 10–12 and 20–22, if the absolute value of an element is smaller than the tolerance τ, the element is then dropped.

3. Concluding remarks

We have proposed two algorithms that determine dynamic sparsity patterns for the FAPINV preconditioners on solving general sparse matrices. In the computation phase, the dropping tolerance has been adaptively determined by the norm of the inverse factors and either the norm of the original factors or the largest value of the original factors. Numerical experiments showed that FAPINV with the proposed dynamic sparsity pattern generates more accurate and robust preconditioner than FAPINV with not only a static sparsity pattern but also other dynamic sparsity pattern (Bollhöfer's) preconditioners do.

References

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