

Modified Tangential Frequency Filtering Decomposition And Its Fourier Analysis

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We consider preconditioning techniques for solving the linear system $\mathbf{Ax} = \mathbf{b}$ with

$$\mathbf{A} = \begin{bmatrix} D_1 & U_1 & & & \\ L_1 & D_2 & \ddots & & \\ & \ddots & \ddots & & \\ & & & U_{n_x-1} & \\ & & & L_{n_x-1} & D_{n_x} \end{bmatrix} \in \mathcal{R}^{N \times N}, \quad \mathbf{b} \in \mathcal{R}^N,$$

which often arises from the discretization of many PDEs by finite difference or finite volume schemes with structured grids.

We present a modification of a tangential frequency filtering decomposition (TFFD) preconditioner [1]. The key idea is to add a “lumping term” ch^q whose influence can be determined by means of Fourier analysis [2, 3]. For the standard five-point stencil, we derive an optimal order $q = \frac{4}{3}$, and constant $c = (2\pi)^{\frac{4}{3}}$. With the choice of optimal order of modification, the Fourier results show that the condition number of the preconditioned matrix is asymptotically $\mathcal{O}(h^{-\frac{2}{3}})$, and the spectrum distribution of the preconditioned matrix can be predicted by the Fourier results. The performance of MTTFFD preconditioner is compared with tangential frequency filtering (TFFD) preconditioner on a variety of large sparse matrices arising from the discretization of PDEs with discontinuous coefficients. The numerical results show that the MTTFFD preconditioner is much more efficient than the TFFD preconditioner.

References

- [1] Y. Achdou, F. Nataf, Low frequency tangential filtering decomposition. *Numer. Linear Algebra Appl.*, 14:129–147, 2007.
- [2] T. F. Chan, H. C. Elman, Fourier analysis of iterative methods for elliptic problems, *SIAM Review.*, 31: 20–49, 1989.
- [3] T. F. Chan, Fourier analysis of relaxed incomplete factorization preconditioners, *SIAM J. Sci. Comput.*, 12: 668–690, 1991.