A Modified Preconditioned Group $(I + \overline{S})$ Iterative Method In The Solution Of Elliptic PDEs

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Many physical phenomena in static field problems particularly in the electromagnetic field and the incompressible potential flow field are described by elliptic partial differential equations (pdes). Improved techniques using explicit group methods derived from the standard and skewed (rotated) finite difference operators have been developed over the last few years in solving the linear system that arise from the discretization of these elliptic pdes (Ali et al., 2004; Evans and Yousif, 1990; Othman and Abdullah, 2000; Yousif and Evans, 1995). The convergence rates of these iterative methods depend on the spectral properties of the coefficient matrices resulted from these group discretization formulas. The formulation of suitable preconditioners which can improve the convergence rates of these iterative schemes are crucial to the development of these new group methods. This paper is concerned with the application of suitable preconditioning technique to a recently developed scheme, the Modified Explicit Decoupled Group (MEDG) iterative method due to Ali and Ng (2008), for solving the two dimensional elliptic pdes. This method was derived from a skewed (rotated) five-point finite difference discretisation which results in a reduced system with lower computational complexity compared to schemes derived from the standard five-point difference approximation. The application of a modified $(I + \overline{S})$-type block preconditioning matrix to the linear system that arise from this group iterative scheme is discussed. The matrix $\overline{S}$ is obtained by taking the first upper diagonal groups of iteration matrix of the original system. Numerical experiments are conducted on each developed non-preconditioned and preconditioned schemes for comparison purposes. The results show the improvements on the convergence rate and the efficiency of the newly formulated preconditioned iterative schemes.

Key words and phrases. Preconditioning method, Modified Explicit Decoupled Group (MEDG) method, Successive Over-Relaxation (SOR) method.

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