Preconditioners For The Discrete Thin Plate Spline Method

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Data fitting is an integral part of a number of applications including data mining, 3D reconstruction of geometric models, image warping and medical image analysis. A commonly used method for fitting functions to data is the thin-plate spline method. This method is popular because it is not sensitive to noise in the data.

Traditional thin plate splines use radial basis functions that produce dense linear system of equations whose size increases with the number of data points. This limits the use of such techniques. several works, but the techniques still lead to complex data structures and the memory requirements increase with the number data points.

In [1, 2, 3] we proposed a discrete thin-plate spline method that uses piecewise functions with local support defined on a finite element mesh. The advantage of using functions with local support is that the dimension of the resulting system of sparse equations depends only on the number of grid points in the finite element mesh, not the number of data points.

Another advantage is that an iterative solver, such as the conjugate gradient method, can be used to solve the system. However it can be shown that the system of equations are similar to those arising from Tikhonov regularisation, and consequently the equations are ill-conditioned for certain choices of parameters.

We recently formulated a simple preconditioning technique based on the Sherman-Morrison-Woodbury formula. The formula allows us to divide the domain up into regions where there are lots of data points, and the interpolant is well defined, and regions where there are few data points. By using different types of grids in each region we are able to solve the problem for a much wider range of parameters.

References

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