A Block ILU Preconditioner for Computational Electromagnetics Applications

List of authors:

J. Marín [1]

R. Bru [2] N. Malla [3]

In this work we consider the solution of the linear systems arising from electromagentism applications by preconditioned Krylov subspace methods [4]. Simulation of electromagnetic wave propagation phenomena requires the numerical solution of Maxwell's equations which it is often done by means of integral equation methods. The discretization of the integral equations with the boundary element method results in dense linear systems of the form,

$$Ax = b \tag{1}$$

where A has complex entries and it is quite challenging to solve. Sparse approximate inverse preconditioners based on Frobenius norm minimization [3] are quite effective for this type of problems. In [2] a number of preconditioners are compared and it is shown that the SPAI preconditioner performs the best. It is also shown that ILU-type preconditioners fail to produce good convergence results. In this work we experiment with ILU-type preconditioners. Following the work presented in [1], we use graph partioning techniques in order to split the adjacency graph of a sparsified matrix $\hat{A}$ in several parts of almost equal size and with the number of edge cuts as small as possible. The matrix $\hat{A}$ is obtained from A after discarding small entries which correspond to far field interactions. Nodes which are connected with cut edges are removed from the subgraphs and put in a separator set. By numbering the nodes by subgraphs and taking the separator set last, the matrix is permuted into block form,

$$P^T\hat{A}P = \begin{pmatrix} \hat{A}_1 & & & & \hat{B}_1 \\ & \hat{A}_2 & & & \hat{B}_2 \\ & & \ddots & & \vdots \\ & & & \hat{A}_p & \hat{B}_p \\ \hat{C}_1 & \hat{C}_2 & \dots & \hat{C}_p & \hat{A}_S \end{pmatrix}$$

This permuted matrix can be decomposed as,

$$P^T\hat{A}P = \begin{pmatrix} \hat{A}_1 & & & & \hat{B}_1 \\ & \hat{A}_2 & & & \hat{B}_2 \\ & & \ddots & & \vdots \\ & & & \hat{A}_p & \hat{B}_p \\ \hat{C}_1 & \hat{C}_2 & \dots & \hat{C}_p & \hat{A}_S \end{pmatrix} = \begin{pmatrix} \hat{L}_1 & & & & \\ & \hat{L}_2 & & & \\ & & \ddots & & \vdots \\ & & & \hat{L}_p & \\ \hat{F}_1 & \hat{F}_2 & \dots & \hat{F}_p & \hat{L}_S \end{pmatrix} \begin{pmatrix} \hat{U}_1 & & & & \hat{E}_1 \\ & \hat{U}_2 & & & \hat{E}_2 \\ & & \ddots & & \vdots \\ & & & \hat{U}_p & \hat{E}_p \\ & & \dots & & \hat{U}_S \end{pmatrix}$$

[1] Institut de Matemàtica Multidisciplinar, Universitat Politècnica de València
[2] Institut de Matemàtica Multidisciplinar, Universitat Politècnica de València
[3] Institut de Matemàtica Multidisciplinar, Universitat Politècnica de València

where $\hat{A}_i = \hat{L}_i\hat{U}_i$, $\hat{E}_i = \hat{L}_i^{-1}\hat{B}_i$, $\hat{F}_i = \hat{C}_i\hat{U}_i^{-1}$ and $\hat{L}_S$, $\hat{U}_S$ are the triangular factors of the Schur complement matrix

$$\hat{S} = \hat{A}_S - \sum_{i=1}^{p} \hat{C}_i\hat{A}_i^{-1}\hat{B}_i.$$

By computing approximate LU factors for the matrices $\hat{A}_i$ and $\hat{S}$ with an appropiate ILU technique, a two-level ILU precontioner for the system (1) is obtained. The results of the numerical experiments show that the block ILU precontioner proposed is competitive.

References

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