

# Exploiting the Nested Block Structure of DG Matrices: a Block ILU Preconditioner with Deflation and a Spectral Two-Level Strategy

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The main advantages of Discontinuous Galerkin (DG) discretizations for elliptic problems are their flexibility in handling non-matching grids and in designing *hp*-refinement strategies. However, an important drawback is that the resulting linear systems are usually large (due to the large number of degrees of freedom) and ill-conditioned. As a consequence, standard Krylov methods typically require too much computational time to solve these systems.

To increase the practical applicability of DG discretizations, we propose a preconditioning technique that is based on a nested block incomplete LU factorization. Earlier, this type of preconditioner was applied successfully in the context of finite difference discretizations for oil reservoir simulation [1, 2]. Here, we combine their ideas with deflation [3] and a spectral two-level strategy to obtain a preconditioner that is suitable for our application, and that efficiently exploits the nested block structure of the coefficient matrix. Various symmetric test cases demonstrate that this technique can lead to uniform convergence of the conjugate gradient method.

## References

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