Efficient preconditioning of optimality systems

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We propose a rather general preconditioning strategy for the numerical treatment of linear optimality systems (OS) arising in connection with inverse problems for partial differential equations. The preconditioning strategy builds on the framework in [1], which applies to well-posed problems. In the work [2] we explain how the mapping properties of the operators appearing in the OS can be employed to define efficient preconditioners for finite element (FE) approximations of the involved saddle point problem also for problems that are ill-posed. More specifically, it turns out that it is possible to define a scheme such that the number of iterations needed to solve the preconditioned problem is bounded independently of the mesh parameter h, used in the FE discretization, and only increases moderately as the regularization parameter α tends towards zero. In fact, if the associated energy norm is used to define the stopping criterion for the iteration process, then the number of iterations required (in the severely ill-posed case) cannot grow faster than $O((\ln(\alpha))^2)$. This result is obtained by carefully analyzing the properties of the involved operators and thereby revealing the distribution of the eigenvalues of the preconditioned OS. One advantage of our approach is that it only rely on standard elliptic preconditioners based on e.g. multigrid and domain decomposition.

Our theoretical results will be illuminated by a number of numerical experiments.

References

[1] K.-A. Mardal, and R. Winther. Preconditioning discretizations of systems of partial differential equations, *Numerical Linear Algebra with Applications*, In Press, 2010.

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