Preconditioning Eigensolvers for Computing Unstable Modes of Plasma

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Numerical simulations are essential tools in the study of the processes occurring in magnetically confined plasmas, such as the anomalous transport (large amounts of energy and particles are transported out of the hot core zone of the plasma) which is the result of the microinstabilities in the plasma produced by gradients in the temperature, magnetic field and density. These microinstabilities can be characterized by the analysis of the unstable modes in the linearized gyrokinetic problem,

$$\frac{\partial g}{\partial t} = \mathcal{L}[g], \tag{1}$$

where $\mathcal{L}$ is a time independent, complex, non-Hermitian integro-differential operator. Time integration of (1) can only obtain the dominant unstable mode (with the largest growth rate). However, for detailed analyses also the subdominant unstable modes are required, which can be obtained via the partial eigendecomposition of $\mathcal{L}$.

GENE [1] is a massively parallel code for plasma simulations that solves the linear and nonlinear gyrokinetic equations following an Eulerian approach. In GENE, the operator matrix is never computed explicitly, but implemented in a highly parallelized and efficient matrix-free form, allowing to deal with problems of sizes ranging from several hundred thousand for linear simulations up to a few billion for nonlinear problems, for reasonably accurate models.

In GENE, the eigenvalue computations of the linear operator are carried out by means of SLEPc, the Scalable Library for Eigenvalue Problem Computations [2]. SLEPc provides a collection of parallel iterative eigensolvers on top of PETSc [3], some of them allowing the use of a preconditioner for accelerating the convergence.

Due to the difficulty of building an efficient preconditioner based on a dense operator matrix, which is not available in explicit form, previous works addressed the problem without preconditioning. In particular, moderate success was obtained with the Krylov-Schur method combined with the shift-and-invert spectral transformation [4] as well as employing the Jacobi-Davidson method [5]. In this work, we analyze different preconditioning strategies in the context of GENE.

The discretized operator L is obtained with a combination of spectral and finite difference techniques. As a consequence, it can be split into a dense part L^ϕ and a sparse part L^g. The latter matrix has a block diagonal sparse multi-banded structure because the differentiation is done locally.

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This work considers the utilization of preconditioners based on the finite-difference part of the operator, L^g, because it contains a representative part of the full operator and its structure is adequate for building efficient preconditioners. We will present results for the inexact shift-and-invert Krylov-Schur and Jacobi-Davidson methods, with several preconditioners including domain decomposition techniques (block Jacobi and additive Schwarz) with incomplete factorizations in each subdomain. We analyze the robustness and performance of the solvers in realistic plasma containment devices such as tokamaks and stellarators.

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References

[1] T. Dannert and F. Jenko. Gyrokinetic simulation of collisionless trapped-electron mode turbulence. *Physics of Plasmas*, 12(7):072309, 2005.

[2] V. Hernandez, J. E. Roman, and V. Vidal. SLEPc: A scalable and flexible toolkit for the solution of eigenvalue problems. *ACM Trans. Math. Software*, 31(3):351–362, 2005.

[3] Satish Balay, Kris Buschelman, Victor Eijkhout, William Gropp, Dinesh Kaushik, Matt Knepley, Lois Curfman McInnes, Barry Smith, and Hong Zhang. PETSc users manual. Technical Report ANL-95/11 - Revision 3.1, Argonne National Laboratory, 2010.

[4] J. E. Roman, M. Kammerer, F. Merz, and F. Jenko. Fast eigenvalue calculations in a massively parallel plasma turbulence code. *Parallel Comput.*, 36(5-6):339–358, 2010.

[5] E. Romero and J. E. Roman. A parallel implementation of the Jacobi-Davidson eigensolver and its application in a plasma turbulence code. In P. D'Ambra, M. Guarracino, and D. Talia, editors, *Euro-Par 2010, Part II*, volume 6272 of *Lect. Notes Comp. Sci.*, pages 101–112. Springer, 2010.