An incomplete multifrontal method for element-structured linear systems

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In this talk, we will propose an incomplete multifrontal LU-factorization (IMF) that extends supernodal multifrontal methods to incomplete factorizations [3]. IMF can be used as a preconditioner in a Krylov-subspace method to solve large-scale sparse linear systems with an underlying element structure. In these applications, the system's coefficient matrix is a sum of small dense element matrices, i.e.

$$A = \sum_{e \in \mathcal{E}} P_e A_e P_e^T.$$

Herein, $\mathcal{E}$ is a set of *elements* and P_e a mapping from the local system dofs to the global dofs. Such systems arise e.g. from a finite element discretization of a partial differential equation. The fact that the element matrices are dense is exploited to increase the computational performance and the robustness of the factorization (through partial pivoting within the dense matrices). We compare IMF with the, similar, multilevel ARMS [1], the level of fill-in ILU and the threshold-based ILUT [2] preconditioners. We demonstrate that IMF can attain a higher throughput than the aforementioned preconditioners, by exploiting the dense block structure in element-structured linear systems. IMF is demonstrated to be effective on linear systems derived from some incompressible flow simulation model problems, outperforming the aforementioned preconditioners by one order-of-magnitude in one instance. The preconditioner was also applied to solve general sparse systems, without an underlying element structure. It is shown to be effective and robust on some matrices from the University of Florida sparse matrix collection and the Matrix Market, provided that an artificial element structure can be extracted that is similar to a finite element discretization.

References

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[2] Y. Saad, *ILUT: a dual threshold incomplete ILU factorization*, Numerical Linear Algebra with Applications, 1 (1994), pp. 387–402.

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