# Comparing multisection and recursive bisection partitions in low-rank representations of BEM matrices from electromechanics

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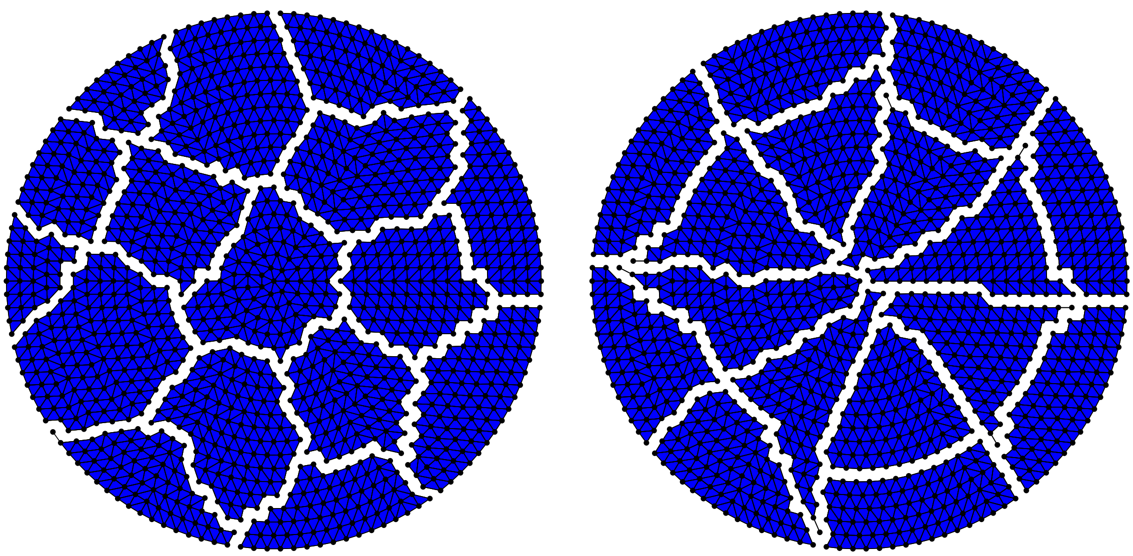
A linear system from a boundary element formulation (BEM) generates dense matrices. However, they can often be permuted and transformed into a data efficient representation using *domain decomposition*.

We search for a partition of the degrees of freedom into *subdomains*. When we create a block matrix using this partition, an off-diagonal block, which represents coupling between two different domains, while dense, may have small numerical rank. The storage for the off-diagonal blocks can be greatly reduced. For some of our larger problems, we have notices storage reductions of 98% (using 1/50-th of the dense storage).

Figure 1: Multisection on (MS) left, recursive bisection (RBS) on right. Each has sixteen subdomains, MS has 425 vertices on interior boundary, RBS has 484.



We consider two types of domain decomposition.

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1. A top-down, divide and conquer scheme we call *recursive bisection*. One looks at a submatrix, divides it into two parts, and recurses on each part if large enough.
2. A bottom-up, create all domains simultaneous scheme we call *multisection*. It is also known as a $k$-way method. The idea is to identify a set of center points. For each domain, we grow and adjust the vertices in a domain around its center point.

The two methods can give partitions with different properties that are visually apparent in Figure 1 for a triangulation of a unit disk.

- Domains in the MS partition seem more rounded than those in RBS.
- Interfaces between domains want to be smooth, but the RBS partition has smoothness at larger scales.
- With our implementations it is possible to finely balance the weights of the domains with RBS, but less so with MS.
- Consistently we see a sizeable difference in the number of interior boundary nodes (those adjacent to a vertex in a different domain). MS outperforms RBS with this metric.

The true judgment of a domain decomposition is how it works with an application. For example, if we were to consider a finite element formulation of the unit disk in Figure 1, we would have a sparse matrix. The number of interior boundary vertices is related to the amount of information communicated among the processors during a matrix-vector multiply.

A BEM matrix is dense, so large communication is expected. We want to reduce the storage for the matrix without sacrificing accuracy. Intuitively, we want subdomains to be compact, with a small aspect ratio. The MS partition appears to be better suited in this regard.

However, a low-rank approximation of a submatrix is best when the submatrix is square. In our context, this means the domains are equally sized. In this respect, RBS partition appears to be better.

In this talk we will provide results for BEM matrices from an electromechanics application from metal forming. We will explore the effects of the two partitions with respect to domain size and grid size.