Incremental Incomplete LU factorizations with applications to time-dependent PDEs

List of authors:

C. Calgaro [1]

J.-P. Chehab [2]

Y. Saad [3]

A common problem which arises in many complex applications is to solve a sequence of linear systems of the form:

$A_k x_k = b_k,$ for $k = 1, 2, ..$

In these applications A_k does not generally change too much from one step k to the next, as it is often the result of a continuous process (e.g. A_k can represent the discretization of some problem at time t_k.) We are faced with the problem of solving each consecutive system effectively, by taking advantage of earlier systems if possible. This problem arises for example in computational fluid dynamics, when the equations change only slightly possibly in some parts of the domain. In such situations it is wastful to recompute entirely any LU or ILU factorizations computed for the previous coefficient matrix.

Though much is known about finding effective preconditioners to solve general sparse linear systems which arise in real-life applications, little has been done so far to address the issue of updating such preconditioners. In our presentation we will consider a number of techniques for computing incremental ILU factorizations.

We will also discuss

the mathematical properties of the new methods as well as of

[1] Université Sciences et Technologies de Lille,
Laboratoire Paul Painlevé, UMR 8524, France and INRIA Lille Nord Europe, EPI SIMPAF, France. `e-mail:caterina.calgaro@univ-lille1.fr`

[2] Université de Picardie Jules Verne,
LAMFA, UMR 6140, Amiens, France and INRIA Lille Nord Europe, EPI SIMPAF, France. `e-mail: jean-paul.chehab@u-picardie.fr`

[3] University of Minnesota, Computer Science, USA
`e-mail: saad@cs.umn.edu`

their implementation: efficient practical implementations require fast sparse solutions of sparse triangular systems. To this end, use the tools developed by [1, 6] for fast (sparse) solutions of sparse triangular systems with spare right hand sides. The algorithm we introduce are based on the following two approaches

- Approximate inverse techniques[4, 5, 7]: we describe a sparse matrix correction technique which computes alternatively improved L and U factors of the factorization. This gives rise to an algorithm referred to as the Minimal Energy Residual descent for LU (MERLU), a descent-type method to drive $\|A - LU\|_F$.
  This technique can be enhanced by dropping strategies and fill-in limitations.
- Alternating correction techniques. In short, these techniques are based on approximately solving the correction equations

  $$(L + X_L)U = R; \quad L(U + X_U) = R$$

  for X_L and X_U, respectively, where R is the residual matrix $R = A - LU$. Then the corrections X_L and X_U are pruned by deleting the upper part from X_L and the lower triangular part from X_L and the strict upper triangular part from X_U. The corresponding iterative method called Iterative Threshold Alternating Lower-Upper (ITALU) algorithm is in effect an alternative fixed point-like iteration.

The methods introduced are tested on a collection of linear problems (3D-convection diffusion, a few matrices from the Matrix Market), as well as on a nonlinear time dependent problem (2D Navier-Stokes equation with Variable density [3]). The new schemes are competitive when compared with existing methods (ILU, ILUT) in terms of fill-in, preconditioning effect (reduction of the number of iteration) and robustness.

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