Numerical experiments on a Factored Approximate Inverse Preconditioner

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We consider an algebraic approach to preconditioning linear systems by computing approximate factors. Let $\mathcal{W}$ and $\mathcal{V}_1$ be sparse matrix subspaces of $\mathbb{C}^{n\times n}$ over $\mathbb{C}$ containing invertible elements and assume that nonsingular elements of $\mathcal{V}_1$ are readily invertible. In addition, let the sparsity structures of $\mathcal{W}$ and $\mathcal{V}_1$ determine their dimension.

To approximately factor the inverse of a large and sparse nonsingular matrix $A \in \mathbb{C}^{n\times n}$ into the product WV_1^{-1}, we consider the problem $AW \approx V_1$ with non-zero matrices $W \in \mathcal{W}$ and $V_1 \in \mathcal{V}_1$ regarded as variables *both*. Denote by P_1 an orthogonal projection onto $\mathcal{V}_1$. We then have the optimality criterion

$$\min_{W\in\mathcal{W},||W||_F=1} ||(I-P_1)AW||_F \tag{1}$$

for generating factors W and $V_1 = P_1AW$. For an algorithm for solving the minimization problem (1) and computing the approximate factors of the inverse, we refer to [1].

In this talk we focus on the choice of subspaces $\mathcal{W}$ and $\mathcal{V}_1$. We also discuss how the quality of the computed preconditioner relates to the conditioning of the matrix V_1 and consider the parallel scalability of the algorithm for computing the approximate factors in practice.

As a preprocessing step, we first use `MC64` to compute diagonal scaling matrices D_1 and D_2 and a permutation Q such that the matrix $A_Q = D_1AD_2Q$ has nonzero diagonal entries [4]. We then apply a symmetric permutation P such that the entries which are large in magnitude are located either in the diagonal blocks or in the block upper triangular part of PA_QP^T. In our experiments, the permutation P is computed with `XPABLO` [5] or with a recent technique for finding strongly connected components of a matrix [3].

The sparsity structure of the subspace $\mathcal{W}$ is determined by the sparsity structure of the powers of the sparsified matrix PA_QP^T, i.e., we use a technique similar to [2]. The sparsity structure of the subspace $\mathcal{V}_1$ is chosen as the sparsity structure of the block diagonal or the block upper triangular part of the permuted matrix PA_QP^T. With these, the algorithm for generating the factors W and V_1 can be interpreted as a process for improving a preconditioner $V_1 = P_1A$.

References

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